

Homework Set #6 – Due Monday, March 3

1. One dimensional scattering. Consider scattering from a potential $V(x) = -g\delta(x)$. If we send in a particle from the left

$$\psi_{\text{inc}}(x) = e^{ikx} \quad x < 0$$

there will be an amplitude for both reflection and transmission. The transmitted part may be written

$$\psi_{\text{trans}} = S(E)e^{ikx} = \sqrt{T}e^{i(kx+\delta)} \quad x > 0$$

where T is the (real) transmission coefficient and δ the phase shift (E is the energy).

- a) Find the transmission coefficient and phase shift and show that T is insensitive to the sign of g . What are the limiting values of δ for $E \rightarrow 0$ and $E \rightarrow \infty$ (consider both positive and negative g)?
 - b) Show that $S(E)$ has a pole for imaginary k , corresponding to the bound state of the delta function potential. Show that this bound state only exists for $g > 0$ by considering the properties of the analytically continued scattering problem.
2. Consider (classical) Rutherford scattering of a particle of mass m and incident energy E , with potential energy $V(r) = C/r$ (C constant).

- a) Derive the relation

$$b = \frac{C}{2E} \cot \frac{\theta}{2}$$

between the impact parameter b and the scattering angle θ . [It may be useful to recall that Keplerian orbits can be written as

$$r(\theta) = \frac{L^2}{mC}(1 + e \cos \theta)^{-1}, \quad e^2 = 1 + \frac{2EL^2}{mC^2}$$

(in polar coordinates, with energy E and angular momentum L) where the eccentricity $e > 1$ for hyperbolic trajectories.]

- b) Derive the differential scattering cross section

$$\frac{d\sigma}{d\Omega} = \frac{C^2}{16E^2} \frac{1}{\sin^4(\theta/2)}$$

[see Merzbacher Exercise 13.1] and show that the total cross section diverges.

3. Merzbacher, Exercise 13.7 and Problem 13.2. Scattering from a translationally invariant potential.