Physics 512 **Winter 2003**

Homework Set #7 – Due Monday, March 10

1. Consider the scattering of a beam of spinless particles of momentum $\hbar k$ initially traveling along the $+\hat{z}$ direction by a potential of the form

$$V(\vec{r}) = V_0\big[\delta(x)\delta(y-b)\delta(z) + \delta(x)\delta(y+b)\delta(z)\big]$$

   a) Calculate the scattering amplitude and differential cross section in the Born approximation.

   b) This potential represents scatterers located at $y=-b$ and $y=+b$. How does the quantum result differ from what one would expect classically?

2. Merzbacher, Exercise 13.11. If $V = C/r^n$, obtain the functional dependence of the Born scattering amplitude on the scattering angle. Discuss the reasonableness of the result qualitatively. What values of n give a meaningful answer?

3. This is based on Sakurai, Chapter 7, Problem 3. [See also Merzbacher Eq. (13.85) and Exercise 13.16; however you should actually work out the relevant partial wave phase shifts.] Consider a potential

$$V = \begin{cases} V_0, & \text{for } r < a; \\ 0, & \text{for } r > a \end{cases}$$

   where V_0 is constant and may be positive or negative.

   a) For $|V_0| \ll E = \hbar^2k^2/2m$ and $ka \ll 1$, the differential cross section is dominated by the s-wave phase shift. Using the method of partial waves, show that in this limit the total cross section is given by

$$\sigma_{\text{tot}} = \left(\frac{16\pi}{9}\right)\frac{m^2V_0^2a^6}{\hbar^4}$$

   b) Suppose the energy is raised slightly. Show that the angular distribution can then be written as

$$\frac{d\sigma}{d\Omega} = A + B\cos\theta$$

   Obtain an approximate expression for B/A.

4. This is essentially Merzbacher, Exercise 13.9. Compute the differential cross section for the potential of problem 3 using the Born approximation.

   a) Show that the differential cross section agrees with the result of problem 3*b*) in the limit $ka \ll 1$.

   b) Evaluate the total cross section in the Born approximation (for any value of ka).