Physics 512 **Winter 2003**

Homework Set #8 – Due Friday, March 14

1. This is based on Sakurai, Chapter 7, Problem 9. Consider scattering by a repulsive δ-shell potential
$$\frac{2mV(r)}{\hbar^2} = \gamma\,\delta(r-a), \qquad \gamma > 0$$
   a) Set up an equation that determines the s-wave phase shift, δ_0, as a function of k (where $E = \hbar^2 k^2/2m$).
   b) Assume that γ is very large
$$\gamma \gg \frac{1}{a}, \qquad \gamma \gg k$$
   Show that if $\tan ka$ is not close to zero, the s-wave phase shift resembles the hard-sphere result, $\delta_0 = -ka$.
   c) Also show that for $\tan ka$ close to (but not exactly equal to) zero, resonance behavior is possible; that is, $\cot\delta_0$ goes through zero from the positive side as k increases (so that δ_0 is increasing as k is increasing). Determine approximately the positions of the resonances keeping terms of order $1/\gamma$.

2. To a first approximation, the potential that a charged particle feels from a hydrogen atom can be thought of as that due to a positive point charge at the origin (the proton) plus a uniform region of negative charge occupying a sphere of radius a_0 (the electron cloud).
   a) Calculate, in the Born approximation, the differential cross section for the scattering of a charged particle from the hydrogen atom as modeled above (neglect recoil of the hydrogen atom).
   b) What is the form of the differential cross section for low energy? Compare with the pure Coulomb cross section.
   c) Show that the differential cross section becomes more and more like a pure Coulomb cross section as the energy of the incident particle increases. Explain why this happens.

3. We wish to find an approximate expression for the s-wave phase shift, δ_0, for scattering of low energy particles from the potential
$$V(r) = \frac{C}{r^4}, \qquad C > 0$$
   a) For low energies, $k \approx 0$, the radial Schrödinger equation for $\ell = 0$ may be approximated by dropping the energy:
$$\left[-\frac{1}{r^2}\frac{d}{dr}r^2\frac{d}{dr} + \frac{2mC}{\hbar^2 r^4}\right] R^{<}_{\ell=0}(r) = 0$$

By making the transformation

$$R(r) = \frac{1}{\sqrt{r}}\phi(r), \qquad r = \frac{i}{\hbar}\frac{\sqrt{2mC}}{x}$$

show that the the radial equation may be solved in terms of Bessel functions. Find the appropriate solution, taking into account behavior at $r = 0$.

b) By matching this to $R^{>}_{\ell=0}(r)$ at $r = a$ (where a is chosen such that $\hbar a \gg \sqrt{2mC}$ and $ka \ll 1$), show that

$$\delta_0 = -\frac{k\sqrt{2mC}}{\hbar}$$

(which is independent of a).