Physics 512 **Winter 2003**

Homework Set #10 – Due Friday, March 28

1. This is based on Sakurai, Chapter 5, Problem 28. A hydrogen atom is initially in its ground state ($1s$). At time $t = 0$ we turn on a spatially uniform electric field as follows:

$$\vec{E}(t) = \begin{cases} 0 & t < 0 \\ \mathcal{E}_0 e^{-t/\tau}\hat{z} & t \geq 0 \end{cases}$$

   a) Using first-order time dependent perturbation theory, compute the probability for the atom to be found in each of the three $2p$ states at time $t \gg \tau$. You need not evaluate the radial integrals, but perform all other integrations.

   b) What would happen if instead the atom was in the $2s$ state to begin with?

2. Consider a two-photon transition from an initial state of angular momentum $\ell_1 = 0$ to a lower energy intermediate state of angular momentum $\ell_2 = 1$ to the ground state of the system with angular momentum $\ell_3 = 0$. Both photons are emitted via electric dipole transitions.

   a) Using second order time-dependent perturbation theory, show that the amplitude for this transition is proportional to $\hat{\epsilon}_1 \cdot \hat{\epsilon}_2$ where $\hat{\epsilon}_1$ and $\hat{\epsilon}_2$ are the polarization vectors of the two photons (ignore identical particle effects).

   b) Averaging over photon polarizations, show that the probability distribution for the angle θ between the two photons has the form $P(\theta) \sim 1 + \cos^2\theta$.

3. A hydrogen atom initially in its ground state is exposed to a harmonic perturbation:

$$\vec{E}(t) = \mathcal{E}_0 \cos(\omega t)\hat{z} \qquad t \geq 0$$

   Calculate the rate of ionization of the atom as a function of ω.

4. Given two distinguishable spin-1 particles with vanishing orbital angular momenta, one can form states of total angular momentum $j = 0$, 1 and 2. Now suppose the two particles are identical. What restrictions do we get? What about two spin-2 particles? What is the general rule for allowed values of j? [See Sakurai, Chapter 6, Problem 2].