Physics 512 **Winter 2003**

Homework Set #11 – Due Friday, April 4

1. This is similar to Sakurai, Chapter 6, Problem 7. Two identical spin-$\frac{1}{2}$ fermions are placed in a one-dimensional infinite square well of size L

$$V(x) = \begin{cases} \infty, & x < 0 \text{ or } x > L; \\ 0, & 0 < x < L \end{cases}$$

We assume that the two particles interact mutually via a δ-function potential given by

$$V_{\text{int}}(x_1, x_2) = -\lambda\delta(x_1 - x_2)$$

Find the (complete, not just ground state) eigenenergies of this system, including the first order energy shift from V_{int}. Take into account both triplet and singlet spin combinations.

2. In the partial wave expansion, the scattering cross section σ_{tot} may be expressed in terms of the phase shifts δ_ℓ.

   a) Show that, for the scattering of two identical spinless bosons, the cross section is given by

$$\sigma_{\text{tot}} = \frac{16\pi}{k^2} \sum_{\ell \text{ even}} (2\ell + 1) \sin^2 \delta_\ell$$

   b) What is the expression for the cross section for two identical spin-$\frac{1}{2}$ fermions?

3. This is based on Merzbacher, Exercise 23.9.

   a) Work out the equal-time commutation relations between the components of $\vec{A}(\vec{r}, t)$ and $\vec{E}(\vec{r}\,', t)$.

   b) For $\vec{B} = \vec{\nabla} \times \vec{A}$, what does this say about simultaneous measurements of electric and magnetic fields? Note that you may expect to find the derivative of a delta-function to turn up.

4. Sakurai, Chapter 5, Problem 40. A hydrogen atom in the $2p$ state will decay to the $1s$ ground state through spontaneous emission. Compute the rate of spontaneous emission in the electric dipole approximation and show that this predicts a lifetime for the $2p$ state of 1.6×10^{-9} s. [Ignore electron spin].