Physics 512 **Winter 2003**

Homework Set #12 – Due Friday, April 11

1. The differential cross section for the ejection of an electron with momentum $\vec{\mathbf{k}}_f$ by an incident photon of momentum $\vec{k}$ ($\omega = c|\vec{k}\,|$) and polarization $\hat{\epsilon}$ (the photoelectric effect) may be written as

$$\frac{d\sigma}{d\Omega} = \frac{\alpha|\vec{\mathbf{k}}_f|}{2\pi m\hbar\omega} L^3 |\langle f|e^{i\vec{k}\cdot\vec{r}}\vec{p}\cdot\hat{\epsilon}|i\rangle|^2$$

where the matrix element refers to the initial (bound) and final (free plane wave) electron states. Derive this expression using the quantum theory of radiation (instead of the classical treatment shown in class).

2. A quantized neutral scalar field (Klein-Gordon field) may be expanded in terms of creation and annihilation operators as follows

$$\phi(\vec{r},t) = \phi^{(+)}(\vec{r},t) + \phi^{(-)}(\vec{r},t) = \sqrt{\hbar c^2}\frac{1}{L^{3/2}}\sum_{\vec{k}}\frac{1}{\sqrt{2\omega}}\left[a(\vec{k}\,)e^{i(\vec{k}\cdot\vec{r}-\omega t)} + a^\dagger(\vec{k}\,)e^{-i(\vec{k}\cdot\vec{r}-\omega t)}\right]$$

As usual, $a(\vec{k}\,)$ and $a^\dagger(\vec{k}\,)$ satisfy the commutation relation $[a(\vec{k}\,), a^\dagger(\vec{k}')] = \delta_{\vec{k},\vec{k}'}$ (all others vanish). In addition, ω satisfies the relativistic energy relation

$$\omega = \sqrt{|c\vec{k}\,|^2 + (mc^2/\hbar)^2}$$

a) Compute the expectation value (two-point function)

$$\langle 0|\phi(\vec{r},t)\phi(\vec{r}\,',t')|0\rangle \equiv \langle \text{vac}|\phi(\vec{r},t)\phi(\vec{r}\,',t')|\text{vac}\rangle$$

You do not have to evaluate the sum over $\vec{k}$.

b) Show that the equal time commutator vanishes

$$\langle 0|[\phi(\vec{r},t),\phi(\vec{r}\,',t')]|0\rangle = 0 \quad \text{for} \quad t = t'$$

3. Merzbacher, Exercise 24.11. Show that the 4×4 matrices $\vec{\Sigma}$ where

$$\vec{\Sigma} = (\Sigma^{23}, \Sigma^{31}, \Sigma^{12}), \qquad \Sigma^{\mu\nu} = \frac{i}{2}[\gamma^\mu, \gamma^\nu]$$

satisfy the usual commutation relations for Pauli spin matrices. Show that in the standard representation

$$\vec{\Sigma} = \begin{pmatrix} \vec{\sigma} & 0 \\ 0 & \vec{\sigma} \end{pmatrix}$$